SciencesPo
DEPARTMENT OF ECONOMICS

ScPoEconometrics

Differences-in-Differences

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Recap from last week

- Applied inference tools to regression analysis
- *Standard error* of regression coefficients
- *Statistical significance* of regression coefficients

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Today: *Differences-in-differences*

- Exploits changes in policy over time that don't affect everyone
- Need to find (or construct) appropriate control group(s)
- *Key assumption:* parallel trends
- *Empirical application*: impact of ***minimum wage*** on ***employment***

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- In this lecture, we will cover a popular and rigorous program evaluation method: **differences-in-differences**.
- Next week we will look at **regression discontinuity designs**.

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DiD Requirements:

- 2 time periods: before and after treatment.
- 2 groups:
  - *control group:* never receives treatment,
  - *treatment group:* initially untreated and then fully treated.
- Under certain assumptions, control group can be used as the counterfactual for treatment group

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- Seminal 1994 paper by prominent labor economists David Card and Alan Krueger entitled "Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania"
- Estimates the effect of an increase in the minimum wage on the employment rate in the fast-food industry. Why this industry?

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Pennsylvania and New Jersey are ***very similar***: similar institutions, similar habits, similar consumers, similar incomes, similar weather, etc.

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Let's take a closer at their data

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  # load package
library(diffindiff)
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ck1994 <- njmin
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```r
ck1994 %>%
  select(sheet,chain,state,observation,empft,emppt) %>
  head()
```

```
## # A tibble: 6 x 6
##   sheet chain  state        observation   empft emppt
##   <chr> <chr>  <chr>        <chr>         <dbl> <dbl>
## 1 46    bk     Pennsylvania February 1992  30    15
## 2 49    kfc    Pennsylvania February 1992   6.5   6.5
## 3 506   kfc    Pennsylvania February 1992   3     7
## 4 56    wendys Pennsylvania February 1992  20    20
## 5 61    wendys Pennsylvania February 1992   6    26
## 6 62    wendys Pennsylvania February 1992   0    31
```

Task 1 (10 minutes)

1. Take a look at the dataset and list the variables. Check the variable definitions with `?njmin`.

2. Tabulate the number of stores by `state` and by survey wave (`observation`). Does it match what's in *Table 1* of the paper?

3. Create a full-time equivalent (FTE) employees variable called `empfte` equal to `empft` + 0.5*`emppt` + `nmgrs`. `empft` and `emppt` correspond respectively to the number of full-time and part-time employees. `nmgrs` corresponds to the number of managers. This is how Card and Krueger compute their full-time equivalent (FTE) employment variable (p.775 of the paper).

4. Compute the average number of FTE employment, average percentage of FT employees (out of the number of FTE employees), and average starting wage (`wage_st`) by state and by survey wave. Compare your results with *Table 2* of the paper.

5. How different are New Jersey and Pennsylvania's fast-food restaurants before the minimum wage increase?

Card and Krueger DiD: Tabular Results

Average Employment Per Store Before and After the Rise in NJ Minimum Wage

Variables	Pennsylvania	New Jersey
FTE employment before	23.33	20.44
FTE employment after	21.17	21.03
Change in mean FTE employment	-2.17	0.59

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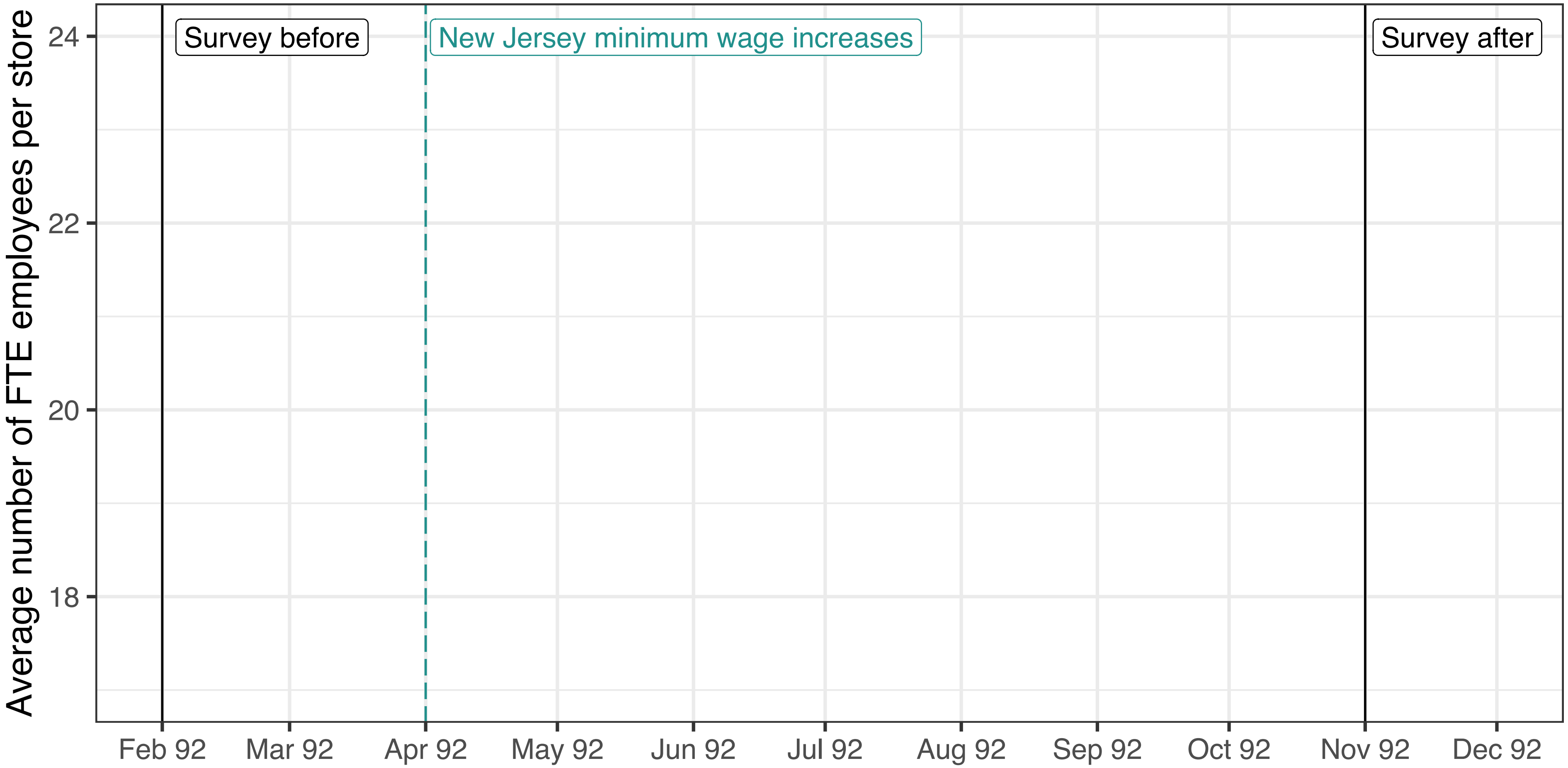
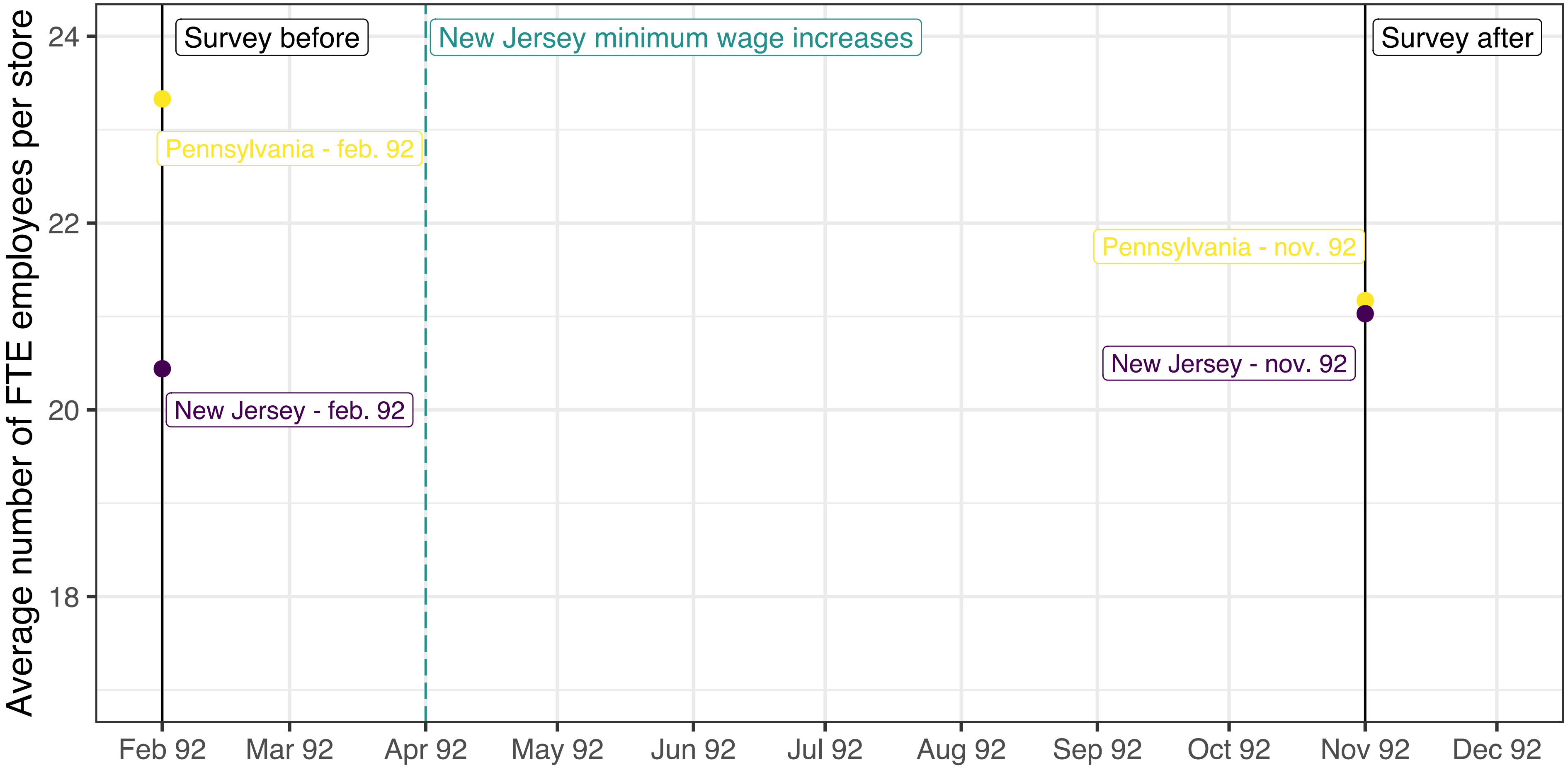
Let's look at these results graphically.

DiD Graphically



DiD Graphically



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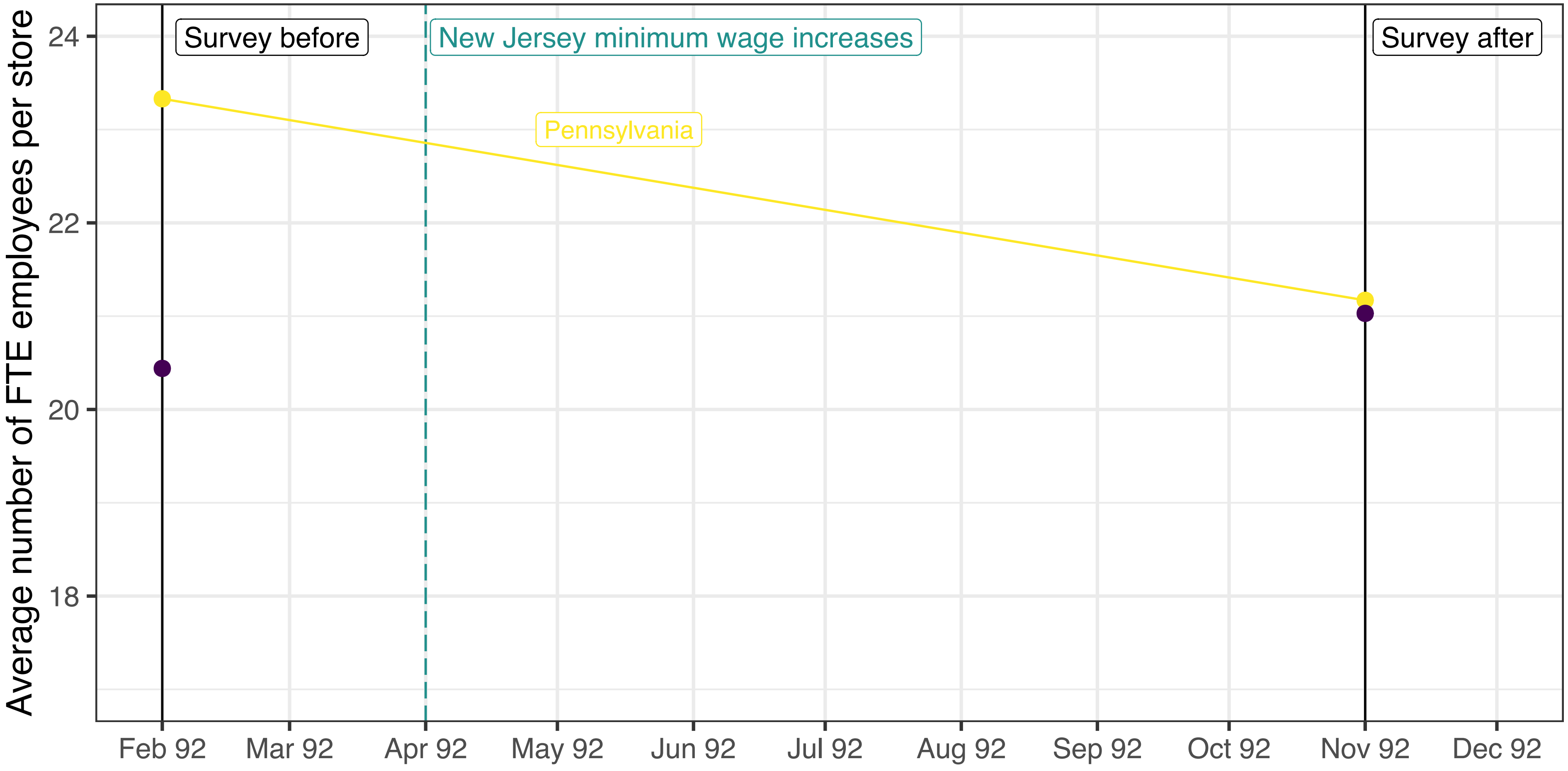

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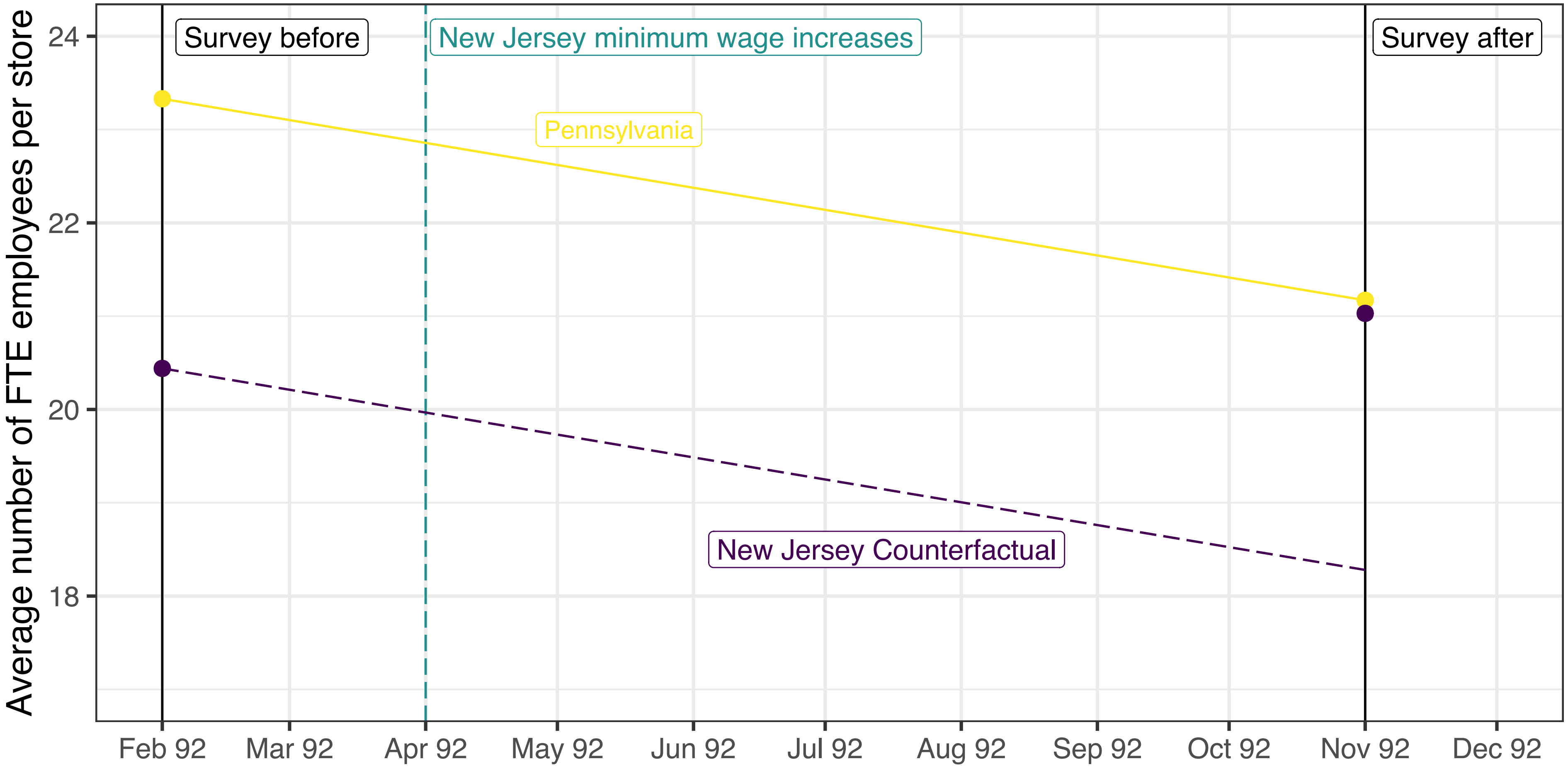

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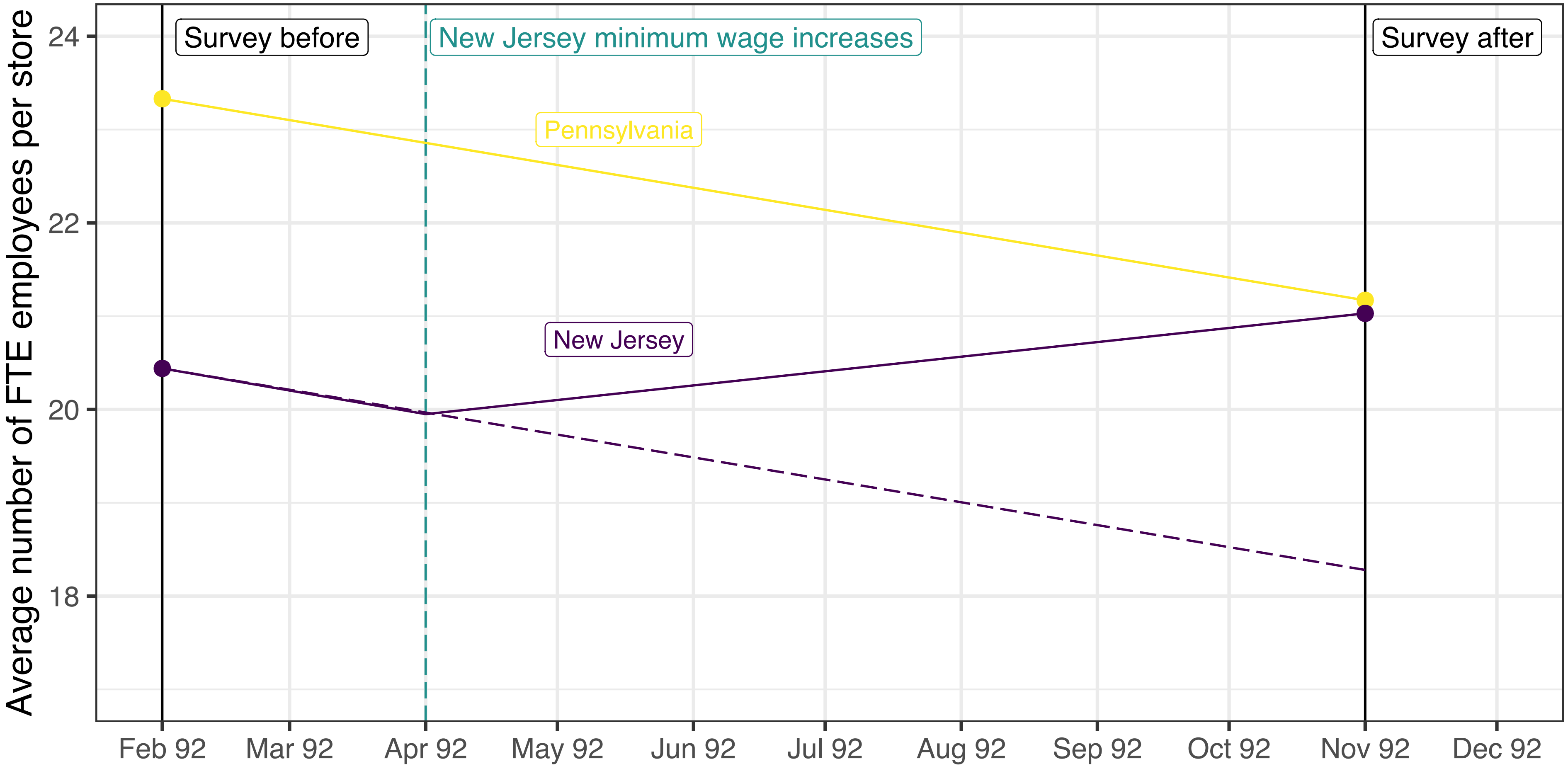

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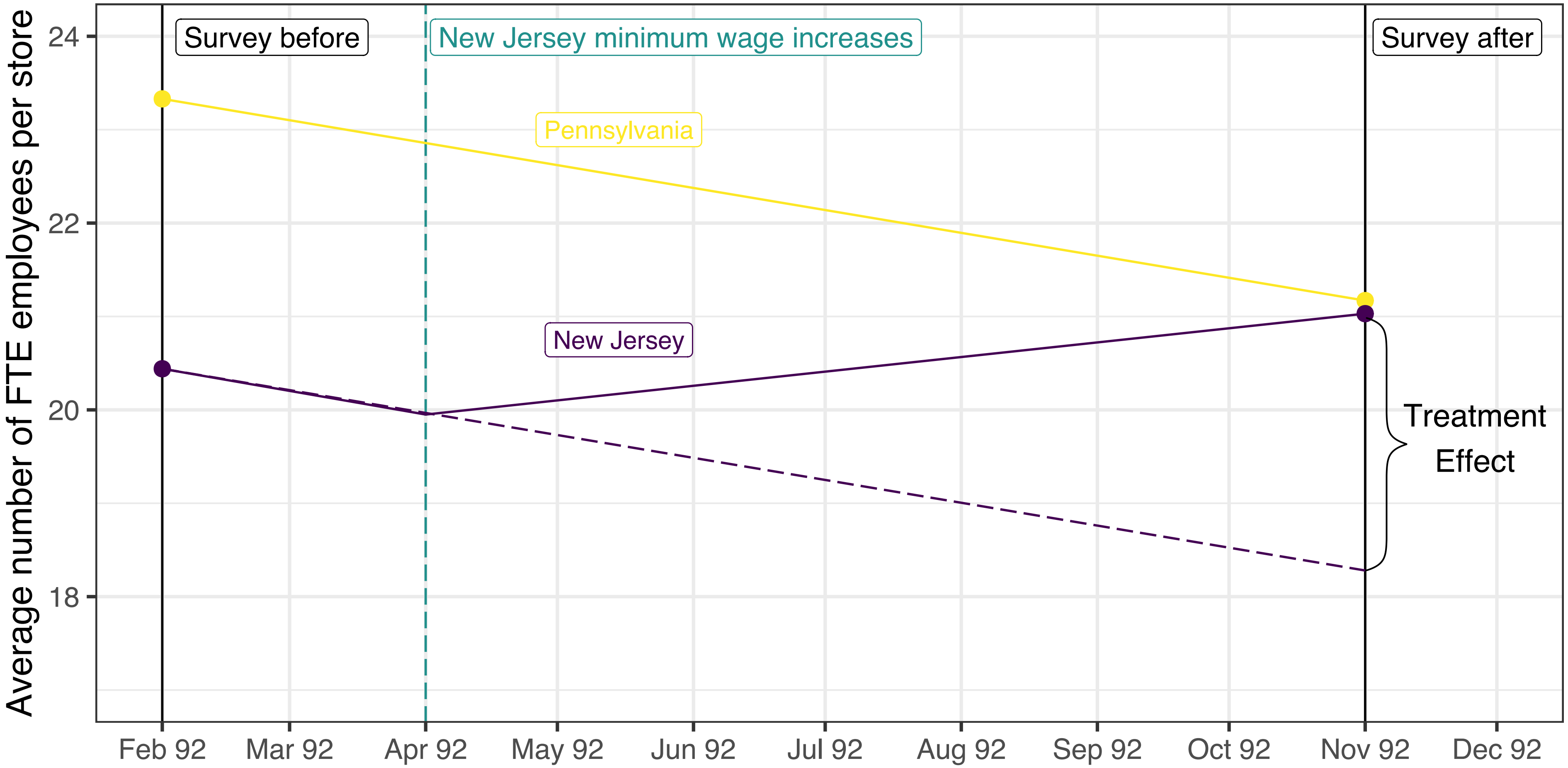
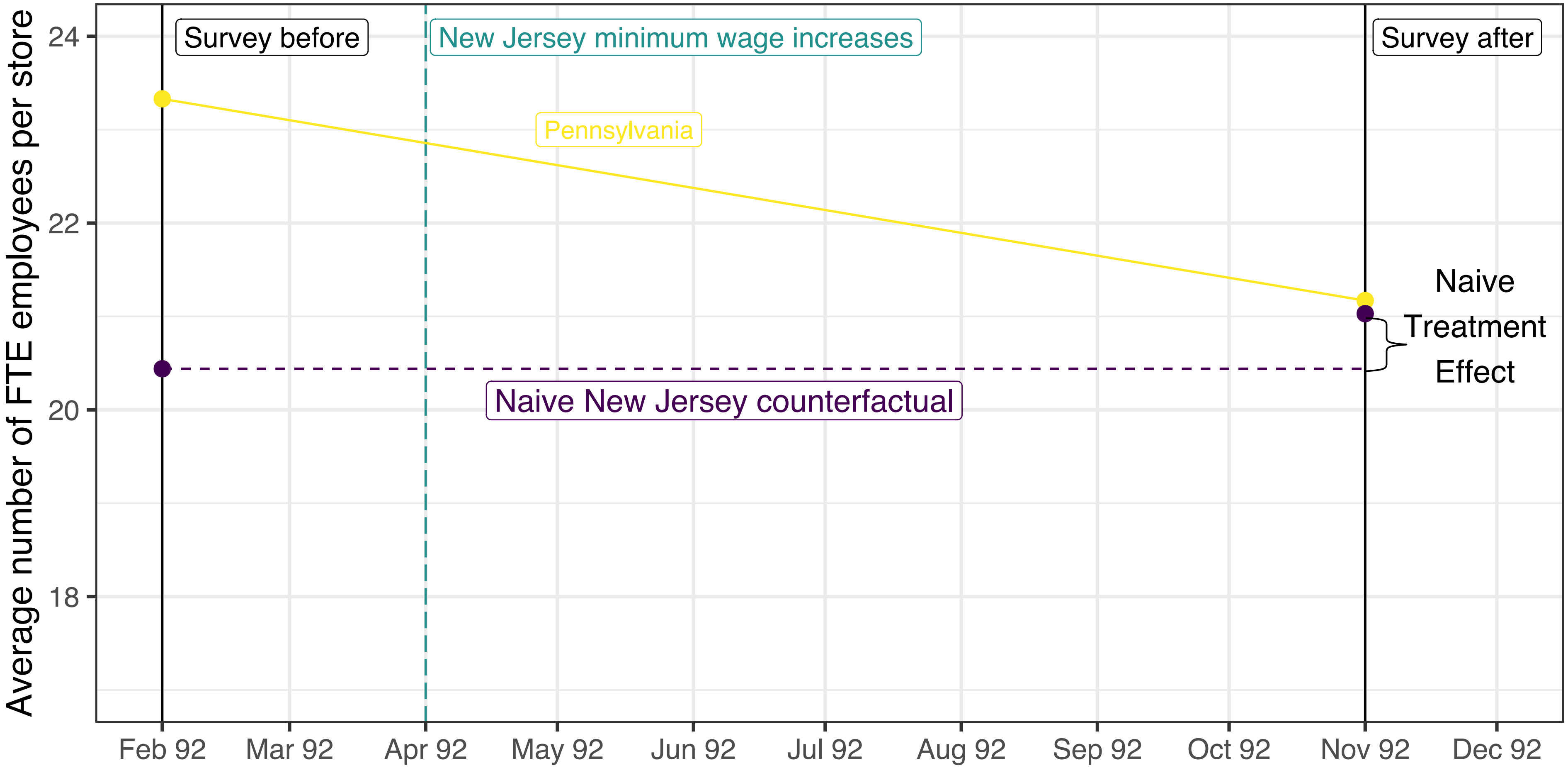


What if we had done a naive after/before comparison?



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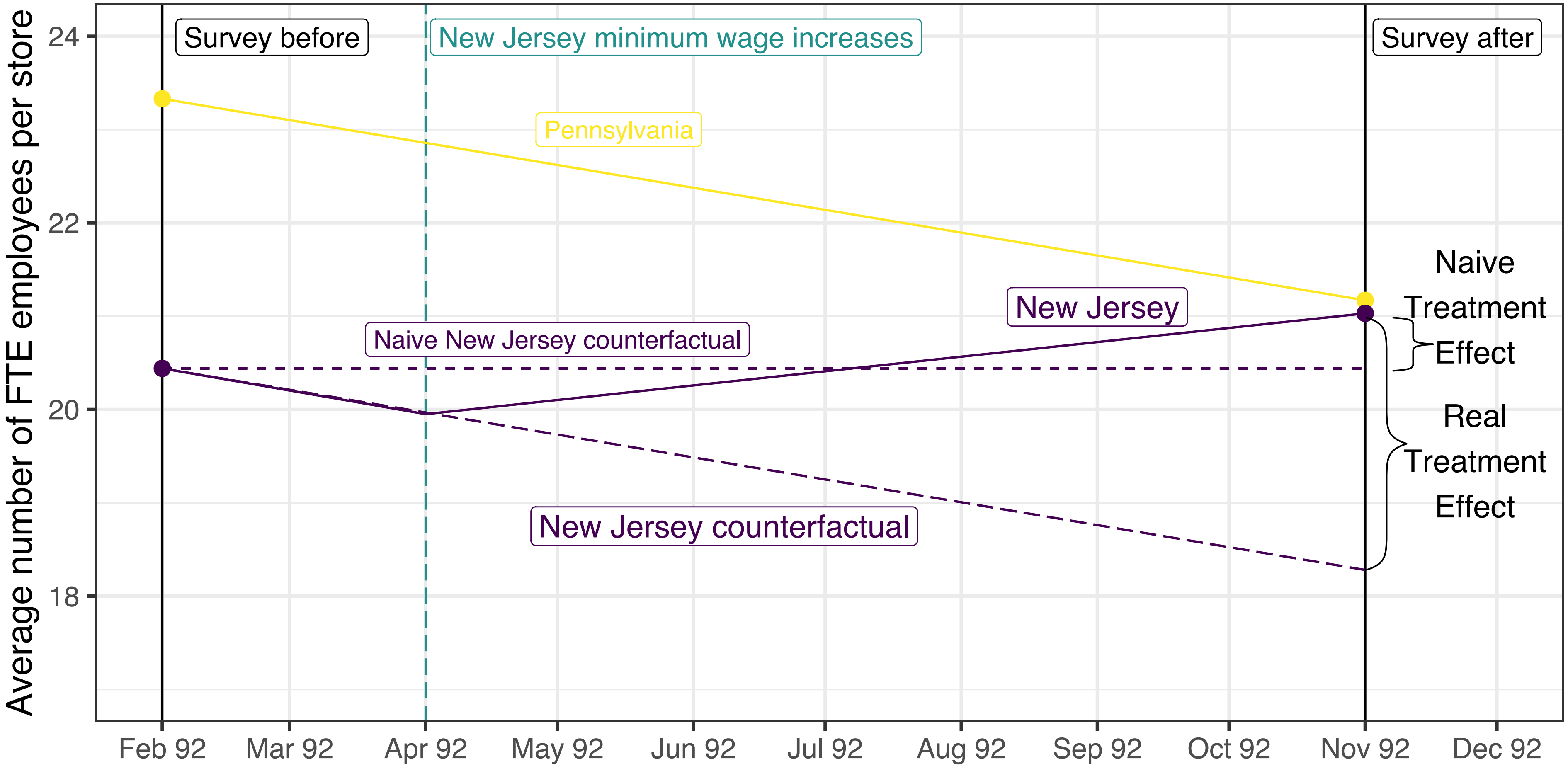

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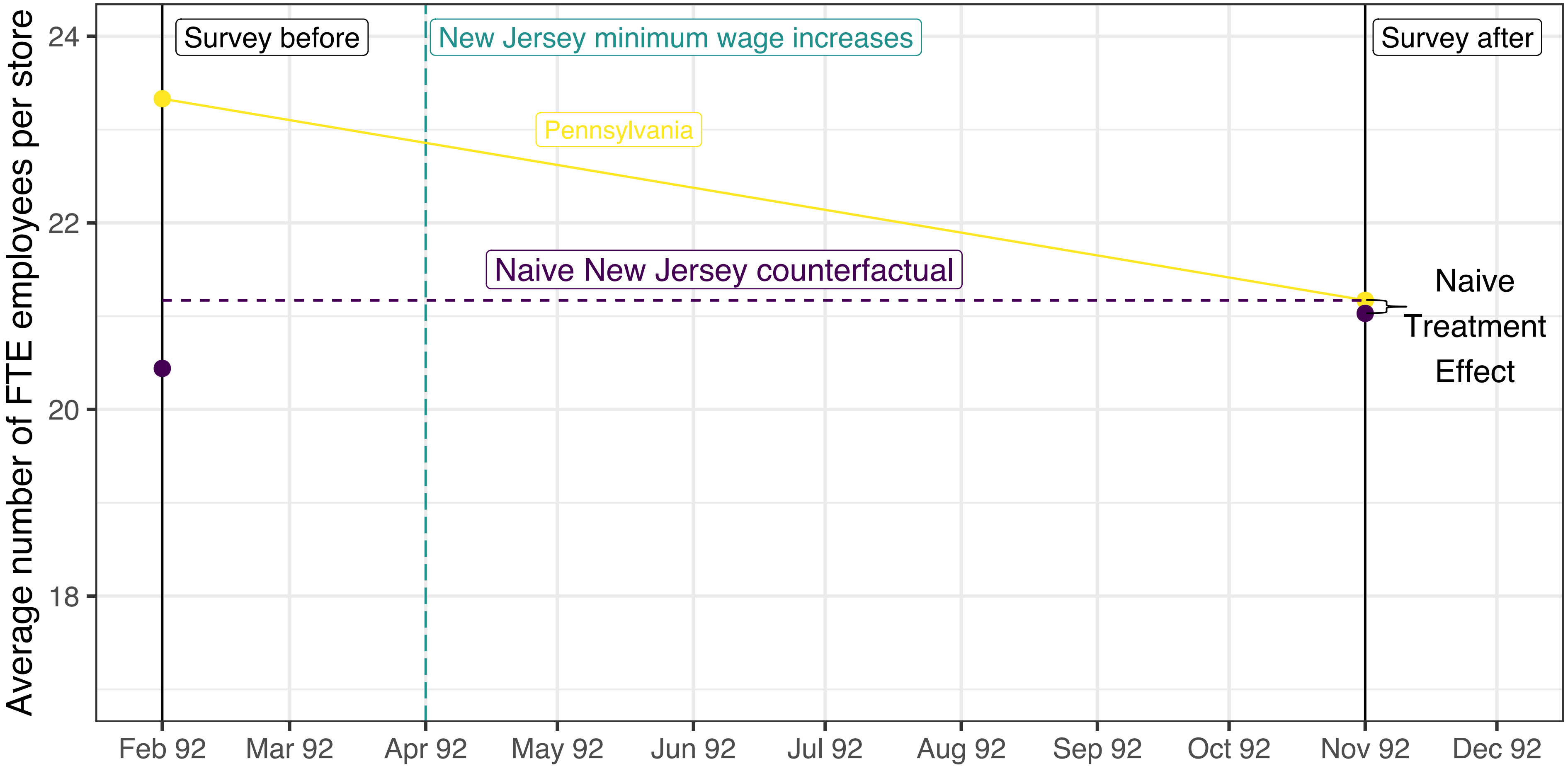

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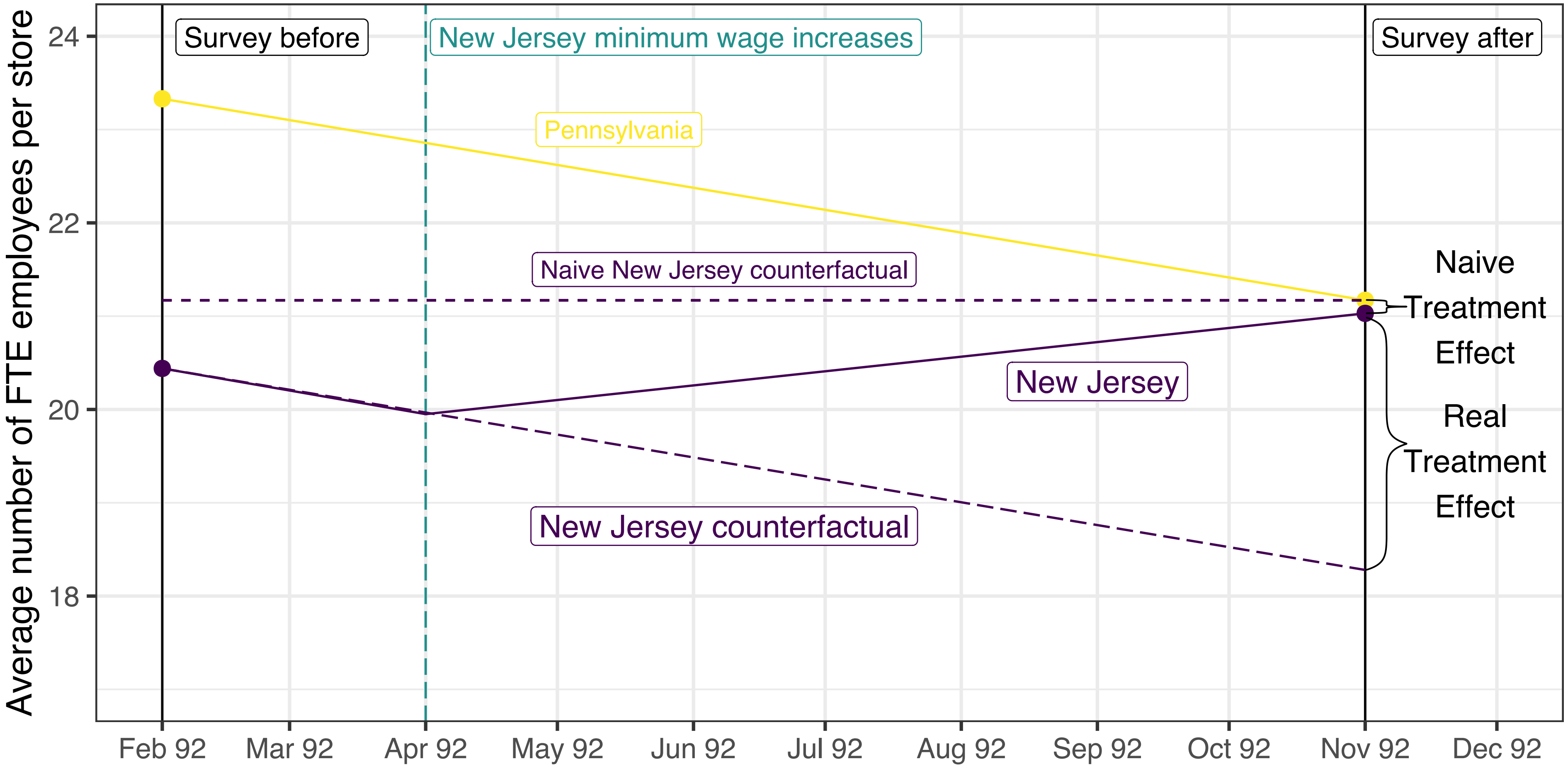

Estimation

DiD in Regression Form

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3. **Interaction term between the two**: $TREAT_s \times POST_t$ 👉 the ***coefficient on this term is the DiD causal effect***!

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- Let's put all these ingredients together:

$$EMP_{st} = \alpha + \beta TREAT_s + \gamma POST_t + \delta(TREAT_s \times POST_t) + \varepsilon_{st}$$

- δ: causal effect of the minimum wage increase on employment

Understanding the Regression

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$$[\mathbb{E}(EMP_{st} \mid TREAT_s = 1, POST_t = 1) - \mathbb{E}(EMP_{st} \mid TREAT_s = 1, POST_t = 0)] -$$
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In table form:

	Pre mean	**Post mean**	**Δ(post - pre)**
Pennsylvania (PA)	α	$\alpha + \gamma$	γ
New Jersey (NJ)	$\alpha + \beta$	$\alpha + \beta + \gamma + \delta$	$\gamma + \delta$
Δ(NJ - PA)	β	$\beta + \delta$	δ

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Δ(NJ - PA)	β	$\beta + \delta$	δ

This table generalizes to other settings by substituting *Pennsylvania* with *Control* and *New Jersey* with *Treatment*

Task 2 (10 minutes)

1. Create a dummy variable, `treat`, equal to `FALSE` if `state` is Pennsylvania and `TRUE` if New Jersey.

2. Create a dummy variable, `post`, equal to `FALSE` if `observation` is February 1992 and `TRUE` otherwise.

3. Estimate the following regression model. Do you obtain the same results as in slide 9?

$$empfte_{st} = \alpha + \beta treat_s + \gamma post_t + \delta(treat_s \times post_t) + \varepsilon_{st}$$

Identifying Assumptions

DiD Crucial Assumption: Parallel Trends

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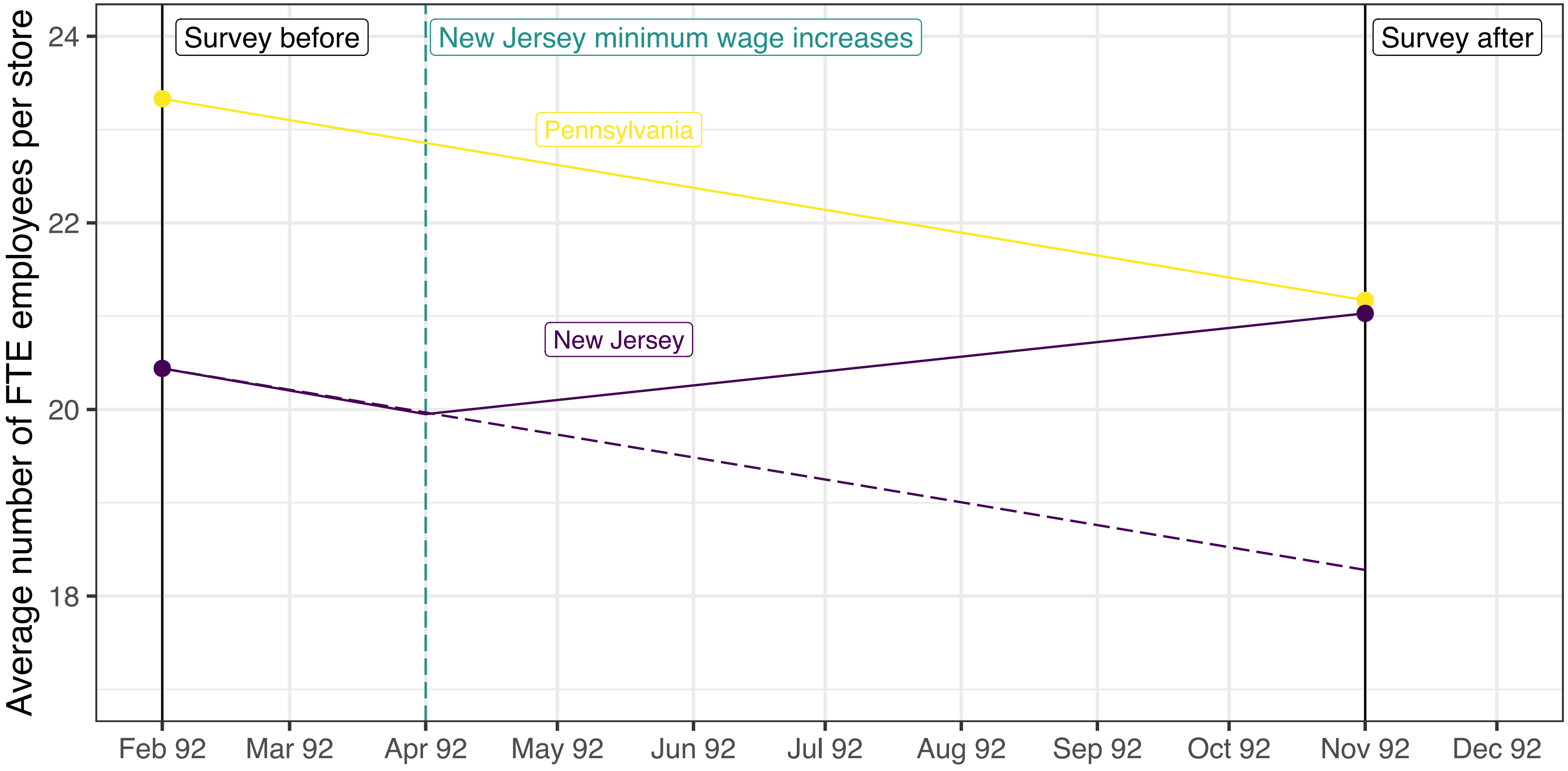
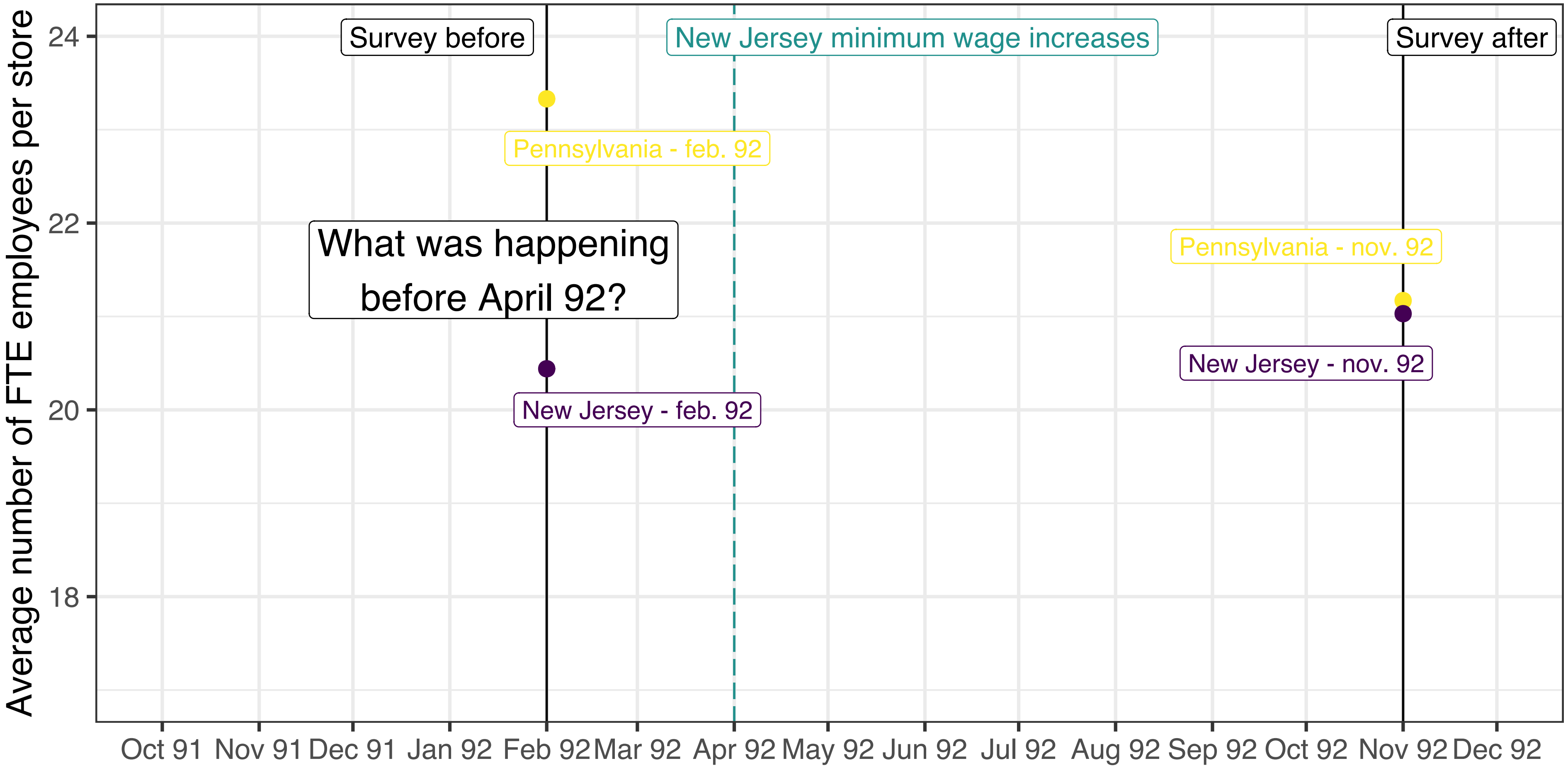
- This assumption states that Pennsylvania's fast-food employment trend between February and November 1992 provides a reliable counterfactual employment trend New Jersey's fast-food industry *would have experienced* had New Jersey not increased its minimum wage.
- Impossible to completely validate or invalidate this assumption.
- *Intuitive check:* compare trends before policy change (and after policy change if no expected medium-term effects)

Parallel Trends: Graphically



Checking the parallel trends assumption



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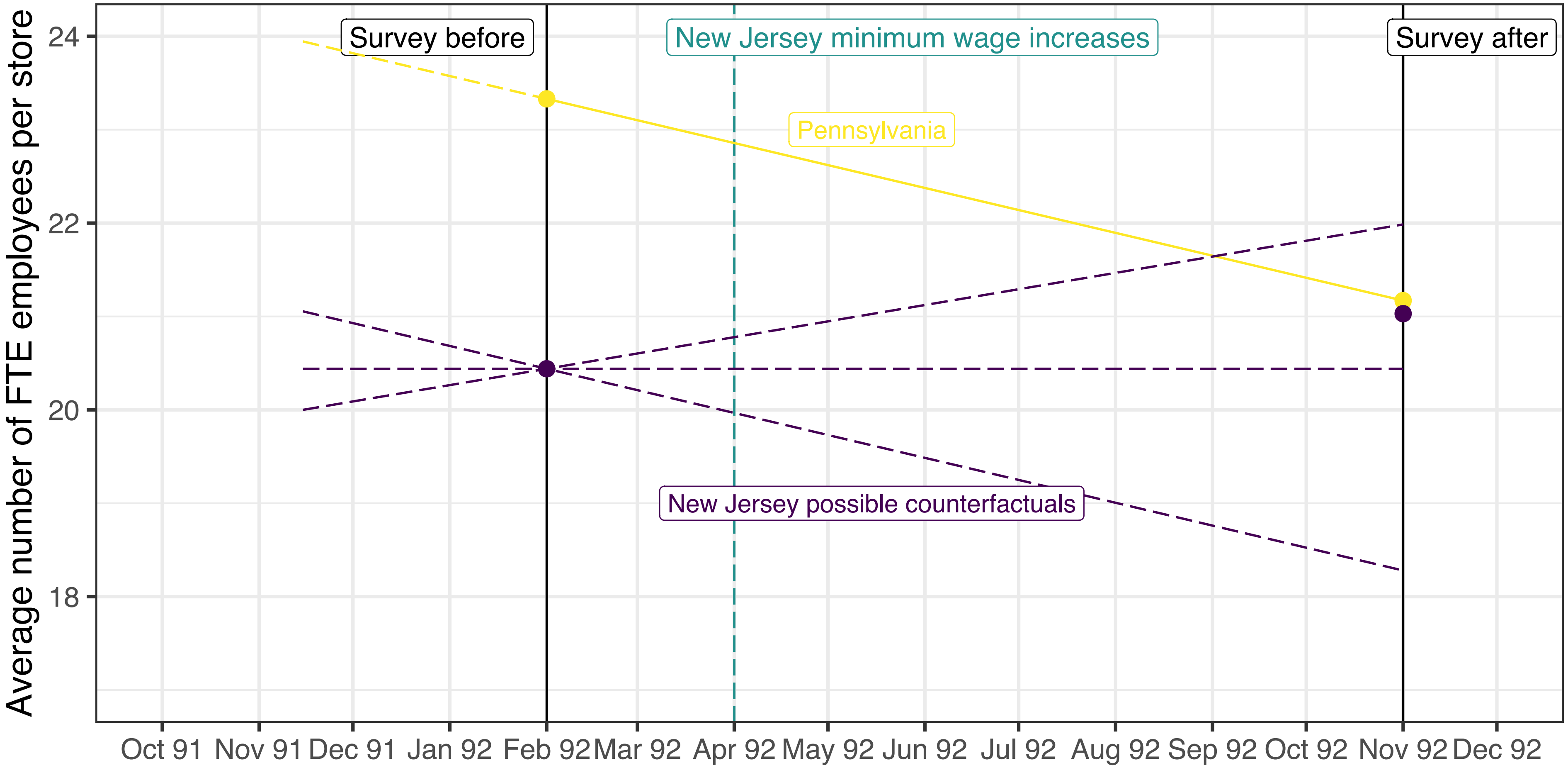
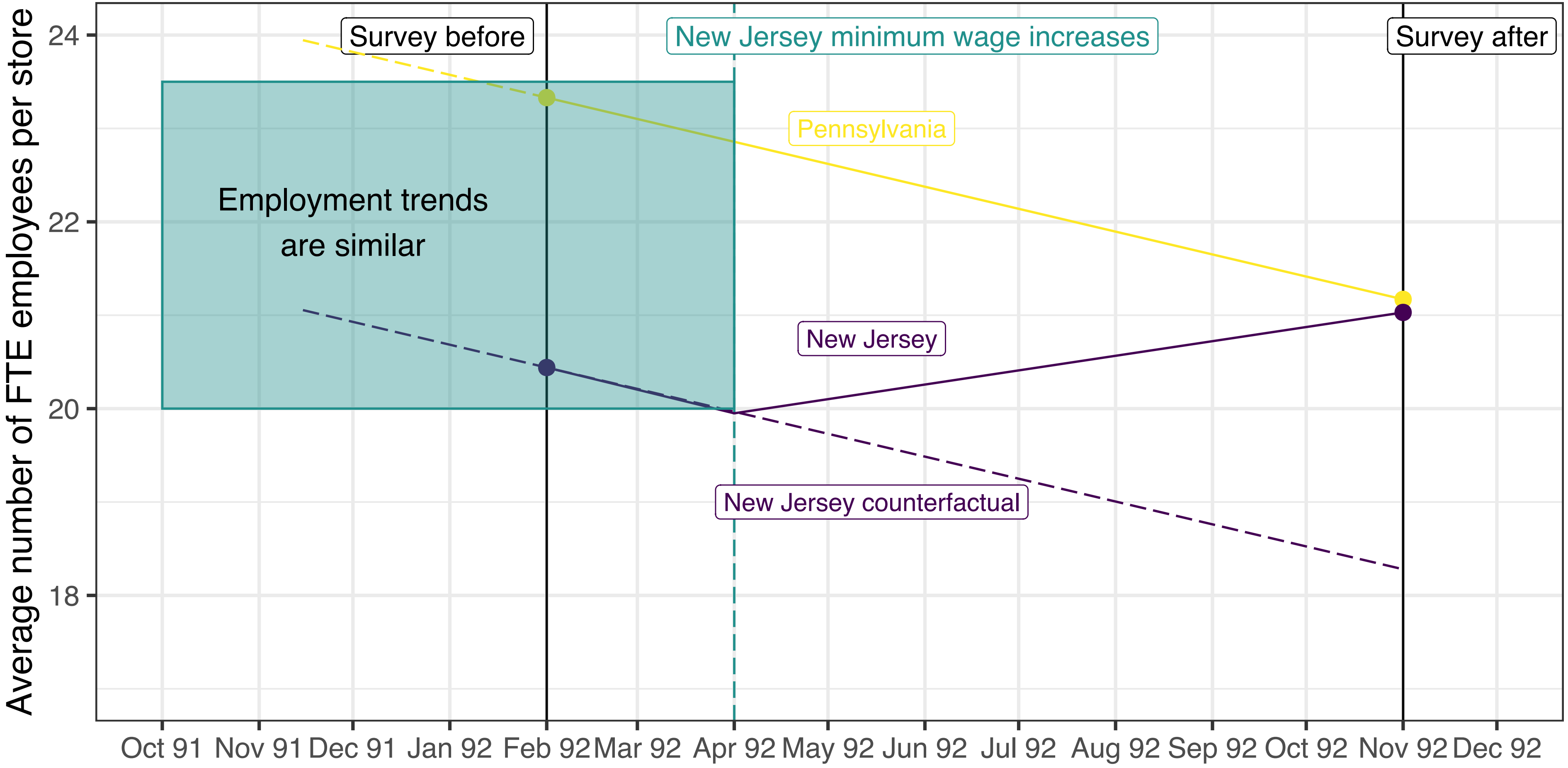


Parallel trends assumption ⟶ Verified ✅



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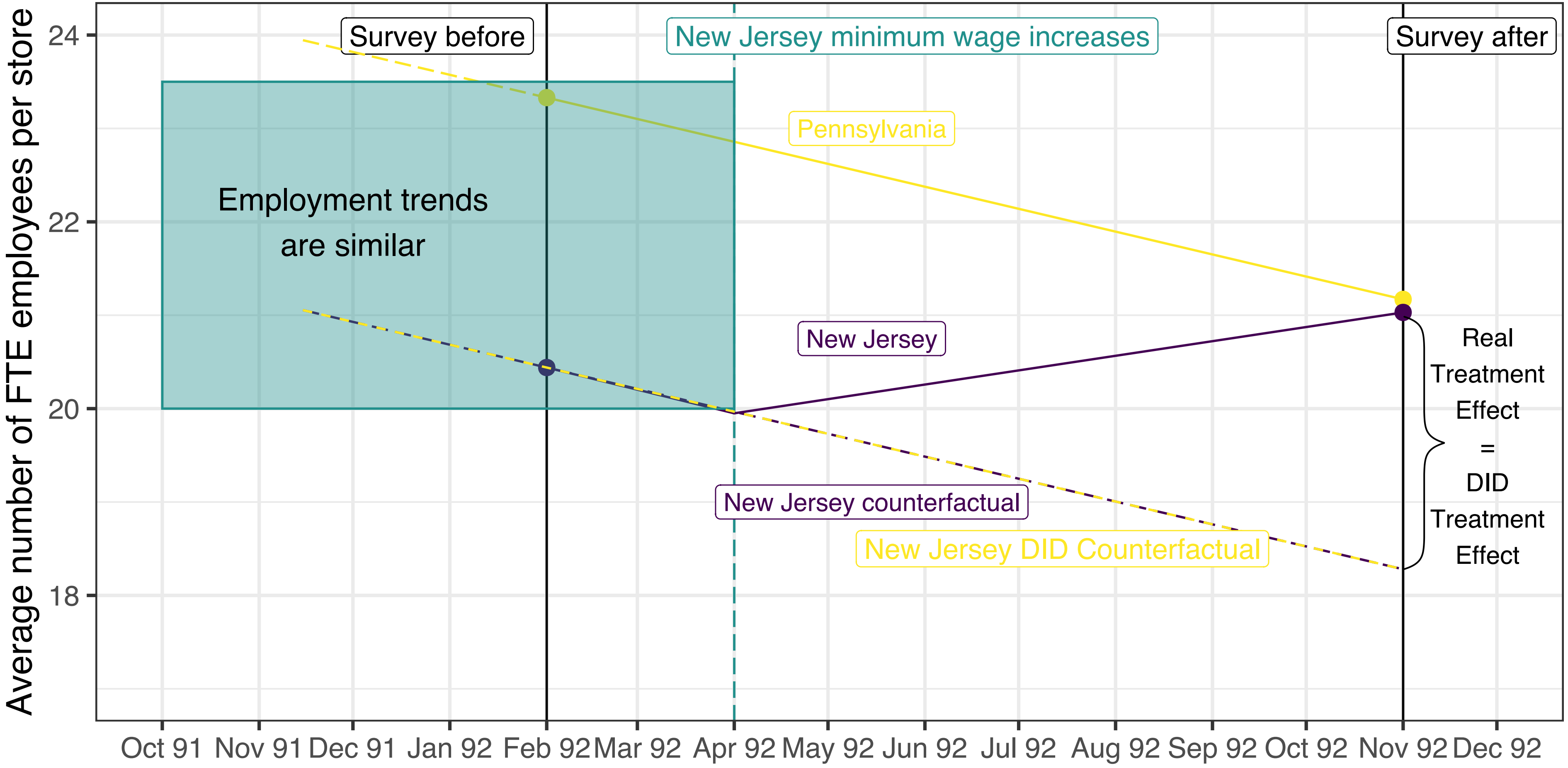

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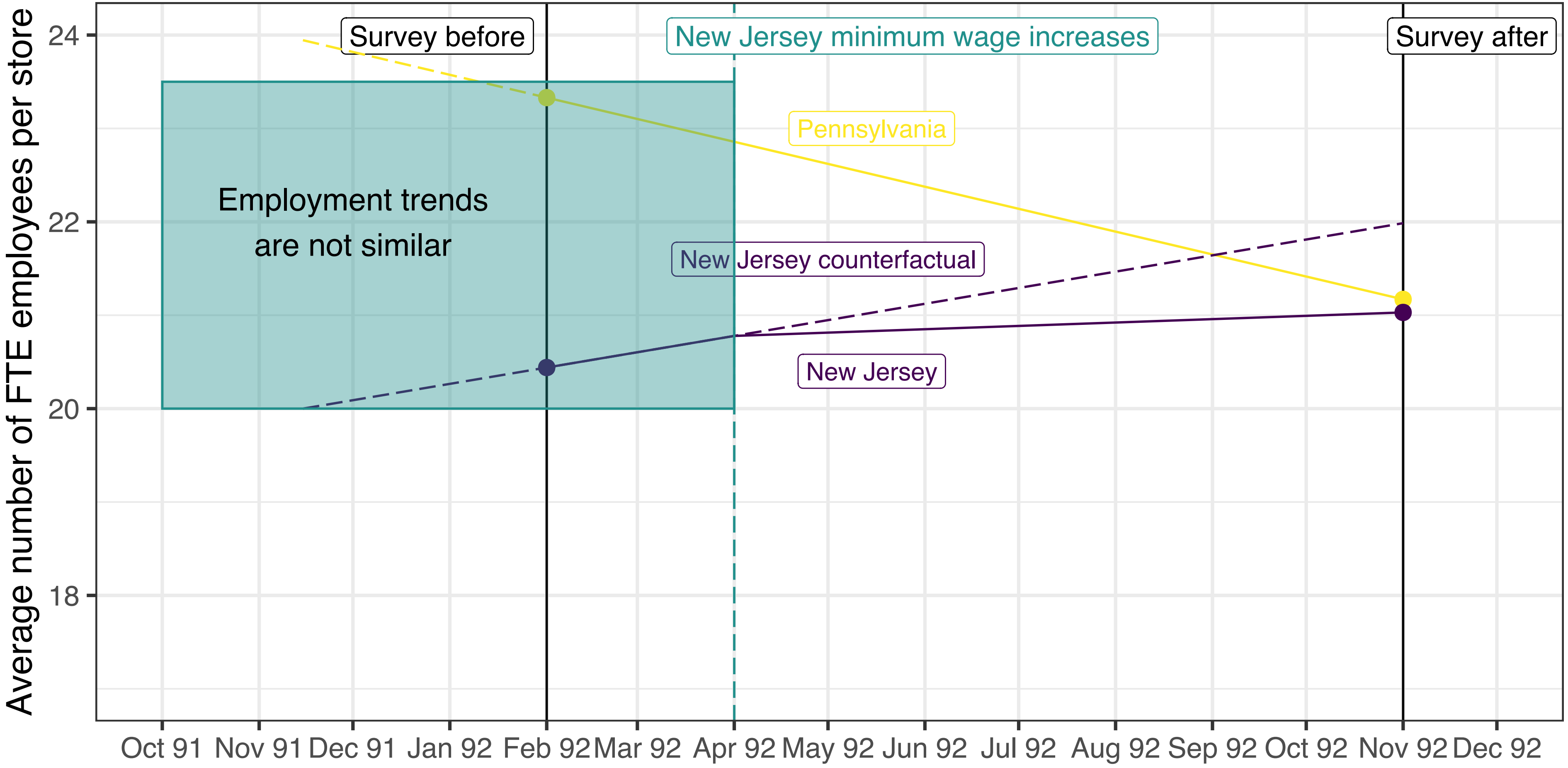
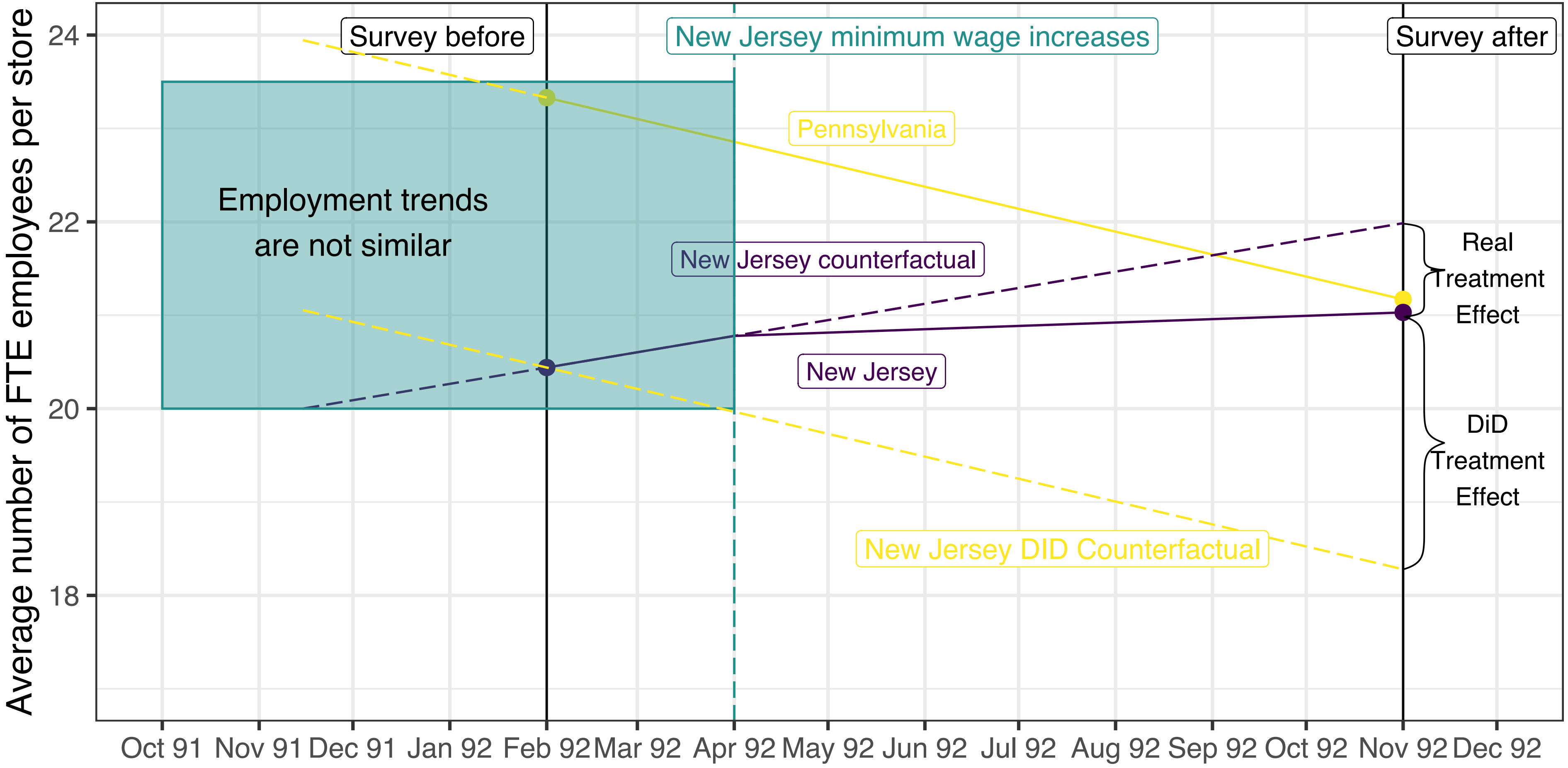
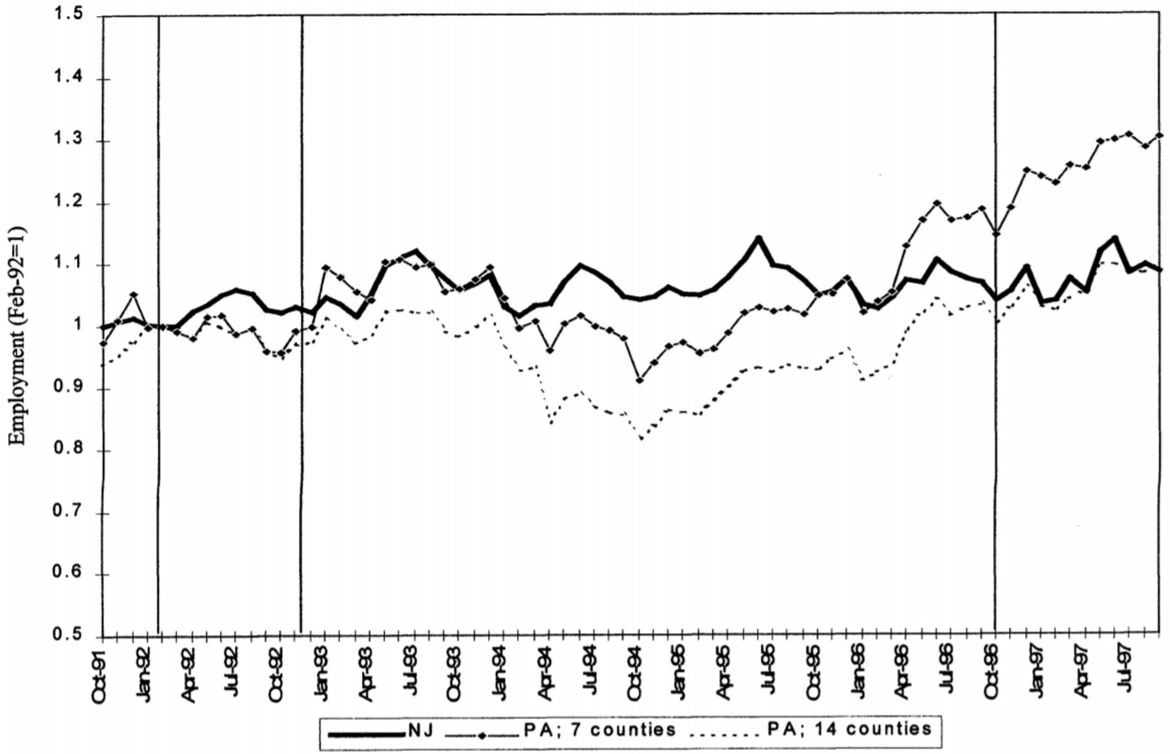


Parallel trends assumption → Not verified ❌



Parallel Trends Assumption: Card and Krueger (2000)

Here is the actual trends for Pennsylvania and New Jersey



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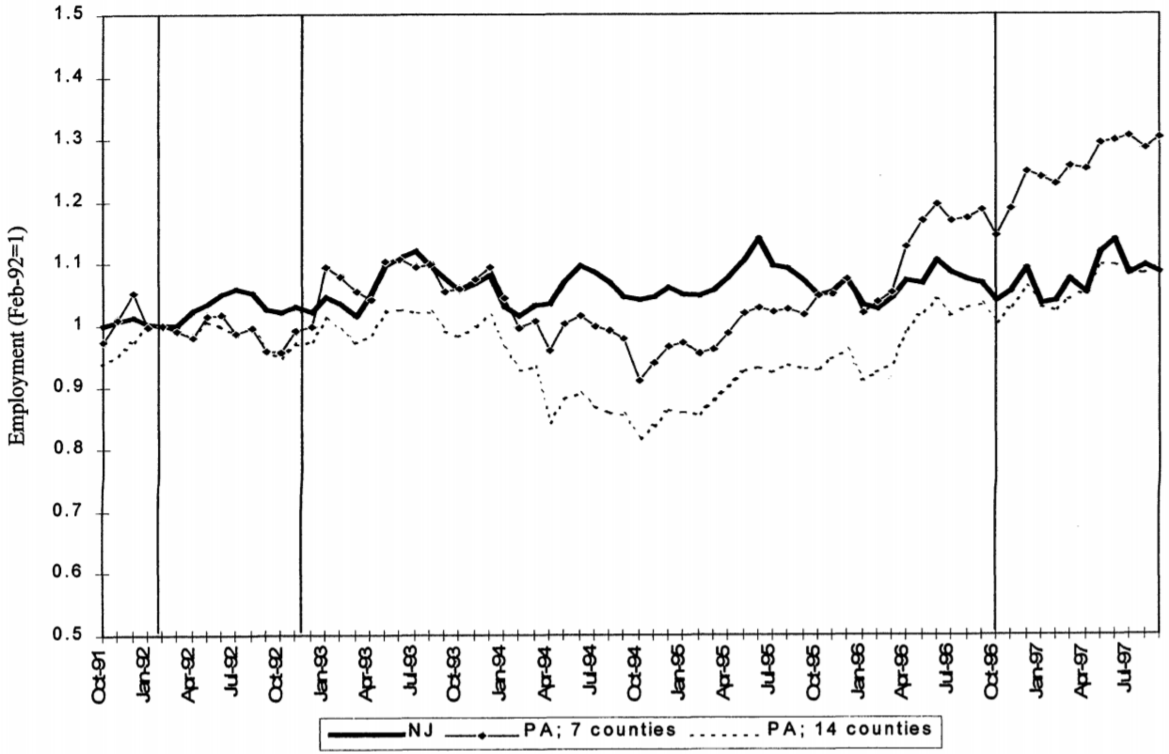

- Is the common trend assumption likely to be verified?

Parallel Trends Assumption: Formally

Let:

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These are potential outcomes, you can only observe one of the two.

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These are potential outcomes, you can only observe one of the two.

The key assumption underlying DiD estimation is that, in the no-treatment state, restaurant i 's outcome in state s at time t is given by:

$$\mathbb{E}[Y_{ist}^0 | s, t] = \gamma_s + \lambda_t$$

2 implicit assumptions:

1. ***Selection bias***: relates to fixed state characteristics (γ)
2. ***Time trend***: same time trend for treatment and control group (λ)

Parallel Trends Assumption: Formally

Outcomes in the comparison group:

$$\mathbb{E}[Y_{ist}|s = \text{Pennsylvania}, t = \text{Feb}] = \gamma_{PA} + \lambda_{Feb}$$

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$$\begin{aligned}
\mathbb{E}[Y_{ist}|s = \text{Pennsylvania}, t = \text{Nov}] - \mathbb{E}[Y_{ist}|s = \text{Pennsylvania}, t = \text{Feb}]&\\
= \gamma_{PA} + \lambda_{Nov} - (\gamma_{PA} + \lambda_{Feb})&\\
= \lambda_{Nov} - \lambda_{Feb}&
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$\rightarrow$ the comparison group allows to estimate the ***time trend***.

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Let δ denote the true impact of the minimum wage increase:

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Parallel Trends Assumption: Formally

Therefore we have:

$$\mathbb{E}[Y_{ist}|s = \text{PA}, t = \text{Nov}] - \mathbb{E}[Y_{ist}|s = \text{PA}, t = \text{Feb}] = \underbrace{\lambda_{Nov} - \lambda_{Feb}}_{\text{time trend}}$$

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$$\mathbb{E}[Y_{ist}|s = \text{NJ}, t = \text{Nov}] - \mathbb{E}[Y_{ist}|s = \text{NJ}, t = \text{Feb}] = \delta + \underbrace{\lambda_{Nov} - \lambda_{Feb}}_{\text{time trend}}$$

$$\begin{aligned} DD &= \mathbb{E}[Y_{ist}|s = \text{NJ}, t = \text{Nov}] - \mathbb{E}[Y_{ist}|s = \text{NJ}, t = \text{Feb}] \\ &\qquad - \Big(\mathbb{E}[Y_{ist}|s = \text{PA}, t = \text{Nov}] - \mathbb{E}[Y_{ist}|s = \text{PA}, t = \text{Feb}]\Big) \\ &= \delta + \lambda_{Nov} - \lambda_{Feb} - (\lambda_{Nov} - \lambda_{Feb}) \\ &= \delta \end{aligned}$$

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END

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