Cournot rationalizability and measurement error

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Western

Before we start

- Excited to be here!
  - Non-traditional tools for applied IO questions
  - Currently getting two kinds of reactions
    - Nah...
    - Cool, but...
- Feedback on:
  - Relevance (motivation, big question)
  - Clear question
  - Where does this paper fit into big question
  - How to sell to applied IO practitioners

Introduction

- Testing firm conduct (competition in pries or quantities, market power) matters (regulation, consumer welfare)
- Standard approach
  - Estimate demand (discrete choice)
  - Given demand, combine with firm competition model
- Assume parametric functional forms and distributions

> Important question: What's the fundamental role of these assumption shaping what we learn from data?

Introduction

- Alternative approach, the Revealed Preference test
- Necessary and sufficient conditions that take the form of a system of inequalities
- Empirically assess the consistency of theoretical models with observational data
- Rely on shape-restrictions, but no parametric assumptions
- However, RP tests are deterministic and could not accommodate ME, until recently.
  - ME is ubiquitous
  - Consumer side: ignoring ME leads to over-rejection of the RP test (Aguiar & Kashaev, 2020)

Where this could go

- Once ME is incorporated into the RP approach, we can invert the test to **derive bounds for latent variables** (Gauthier, 2021)
  - Could say something about the role of assumptions in parametric approach
- Along the way, **non-parametric test for Market Power**
- Set of tools to study models of firm competition **complementary** to standard methods.

This paper

- First step: Shows how to integrate **ME into the RP test** for models of firm competition.

Overview of this paper

- **What's the question?**: Do the observed prices and quantities in the oil industry arise as an equilibrium of the Cournot model when quantities are mismeasured?
- **Why does it matter?**: The OPEC cartel has had a quantity adjusting policy since 1985. Given the cartel's relevance, we would think the Cournot model to be a good fit for their behavior. However, the literature has mixed and inconclusive results.
- **How do I do it?**: I design a stochastic RP test of the Cournot model by introducing ME to Carvajal et al. (*ECTA*, 2013).
- **What do I find?**: In contrast to the deterministic version of the test. the Cournot hypothesis can no longer be rejected once ME in quantities is introduced.

Cournot Rationalizability and ME

- Crude oil industry: The Organization of Petroleum Exporters Countries (OPEC) determines at least twice a year at the "Conference" how to adjust its output (since 1985, before they set their price).
  - Production share (2009): 40.9%
  - Reserves: 85% of world total in 2011 (Wirl, 2015)
  - Reserves-to-production ratio: 90 years 2011 (U.S. 10 years)(Wirl, 2015)
- Prices of all other fuels are linked to the oil price, directly (natural gas contracts in Europe) or indirectly (supply/demand interactions) (Wirl, 2015)
- Mixed and inconclusive results in the literature empirically testing the cartel hypothesis (Griffin, 1985; Griffin and Xiong, 1997; Mason and Polasky, 2005; Smith, 2005; Carvajal et al., 2013; Wirl, 2015; Moghadam, 2021)

Cournot Rationalizability and ME

- Cournot competition: firms compete in quantities and they produce a single homogeneous good with a downward sloping inverse demand function

- Carvajal et al. (2013) developed a RP test for the Cournot model

    - Using 1973-2009 prices and production quantities for OPEC and major Non-OPEC oil producers , the authors reject the Cournot Hypothesis
    - In this setting, ME in quantities could arise from: Consolidation process or Coordination mistakes
    - The result can be driven by the omission of ME in their model
    - Intuition: deterministic convex function rationalizing the data

Setting

- A set of firms $\mathcal{I} = \{1, 2, \ldots, I\}$ produce a single homogeneous good $q_t = \sum_{i \in \mathcal{I}} q_{it}$ with a downward slopping inverse demand function $p(q_t) = p_t$ with $p'(q_t) \leq 0$.
- The data set $\mathcal{O} = \{p_t, (q_{it})_{i \in \mathcal{I}}\}_{t \in \mathcal{T}}$ consists of T observations indexed by $t \in \mathcal{T} = \{1, 2, \ldots, T\}$
- Firms have unobserved convex cost functions and their first derivative at q_{it} is indicated by δ_{it}

Setting

Let $\mathbf{p}_t^* \in \mathbf{P}_t \subseteq \mathbb{R}_{++}^L$ and $\mathbf{q}_t^* \in \mathbf{Q}_t \subseteq \mathbb{R}_{+}^L \setminus \{0\}$ denote random vectors of true prices and true quantities at time t, respectively.

Measurement Error

Measurement error, $\mathbf{w} = (\mathbf{w}_t)_{t\in\mathcal{T}} \in W$, is the difference between the observed random variables and their true values.

$$\mathbf{w} = \begin{pmatrix} \mathbf{w}_t^q \\ \mathbf{w}_t^p \end{pmatrix} = \begin{pmatrix} \mathbf{q}_t - \mathbf{q}_t^* \\ \mathbf{p}_t - \mathbf{p}_t^* \end{pmatrix}, \quad \forall t \in \mathcal{T}$$

Stochastic Cournot Rationalizability

LEMA: The following statements are equivalent:

1. The random array $\{(\mathbf{q}^*_{it})_{i\in\mathcal{I}}, \mathbf{p}^*_t\}_{t\in\mathcal{T}}$ is **stochastic Cournot rationalizable (SCR).**

2. There exists nonnegative random vectors $(\delta_{it})_{(i,t)\in\mathcal{I}\times\mathcal{T}}$ that satisfy:

   i. $\frac{\mathbf{p}^*_t-\delta_{it}}{\mathbf{q}^*_{it}} = \frac{\mathbf{p}^*_t-\delta_{jt}}{\mathbf{q}^*_{jt}} \geq 0$ *(common ratio property)*, and

   ii. $(\delta_{it} - \delta_{is})(\mathbf{q}^*_{it} - \mathbf{q}^*_{is}) \geq 0$ *(co-monotone property)*

   a.s., $\forall t, s \in \mathcal{T}$, and every $i, j \in \mathcal{I}$.

Characterization by moment conditions

- We can summarize the empirical content of the SCR by a set of moment conditions.
- Let $\mathbf{e} = (\delta, \mathbf{w})' \in E|X$ denote the vector of random latent variables and
- $\mathbf{x} = (\mathbf{q}, \mathbf{p})'$ the vector of observed random variables.
- Moreover, let $\mathcal{P}_X$, $\mathcal{P}_{E,X}$, and $\mathcal{P}_{E|X}$ denote the set of all probability measures defined over the support of $\mathbf{x}, (\mathbf{e}, \mathbf{x}),$ and $\mathbf{e}|\mathbf{x}$, respectively.

Characterization by moment conditions

Define the following moment conditions:

$$g_M(\mathbf{x}, \mathbf{e}) = \mathbf{p}_t' \mathbf{w}_t^q,$$

$$g_R(\mathbf{x}, \mathbf{e}) = \mathbf{1}\left[\frac{\mathbf{p}_t - \delta_{it}}{\mathbf{q}_{it} - \mathbf{w}_{it}^q} = \frac{\mathbf{p}_t - \delta_{jt}}{\mathbf{q}_{jt} - \mathbf{w}_{jt}^q} \geq 0\right] - 1,$$

$$g_C(\mathbf{x}, \mathbf{e}) = \mathbf{1}\left[(\delta_{it} - \delta_{is})(\mathbf{q}_{it} - \mathbf{w}_{it}^q - \mathbf{q}_{is} + \mathbf{w}_{is}^q) \geq 0\right] - 1,$$

$\forall i \neq j \in \mathcal{I}, t \neq s \in \mathcal{T}$

Using ELVIS

- We can solve the problem of having a latent random set of variables $\mathbf{e}$ with unknown $\mu \in \mathcal{P}_{E|X}$ using the *Entropic Latent Variable Integration via Simulation* (ELVIS) (Schennach, 2014).
- Intuitively, there might be many possible conditional distributions $\mu \in \mathcal{P}_{E|X}$ of the latent variables that satisfy the moment conditions.
- ELVIS ranks them by entropy and selects the *least favourable*, thus, converting an *existence* problem into an optimization problem.

Using ELVIS

- In practice, I used Markov Chain Monte Carlo (MCMC) methods.
- Intuitively, the RP inequalities define a multidimensional region in the hyperspace.
- First, we find a point inside the region, then, we move randomly inside the object to integrate out the latent variables.

Test statistic

$$TS_n = n \inf_{\gamma \in \mathbb{R}^q} \hat{\tilde{h}}_M(\gamma)' \hat{\tilde{\Omega}}^{-1} \hat{\tilde{h}}_M(\gamma)$$

where $\hat{\tilde{h}}_M(\gamma)$ and $\hat{\tilde{\Omega}}$ are the sample analogues of the maximum-entropy moment and its variance, defined as:

$$\hat{\tilde{h}}_M(\gamma) = \frac{1}{n} \sum_{k=1}^{n} \tilde{h}_M(\mathbf{x}_k; \gamma)$$

$$\hat{\tilde{\Omega}}(\gamma) = \frac{1}{n} \sum_{k=1}^{n} \tilde{h}_M(\mathbf{x}_k; \gamma) \tilde{h}_M(\mathbf{x}_k; \gamma)' - \tilde{h}_M(\gamma) \tilde{h}_M(\gamma)'$$

Test statistic

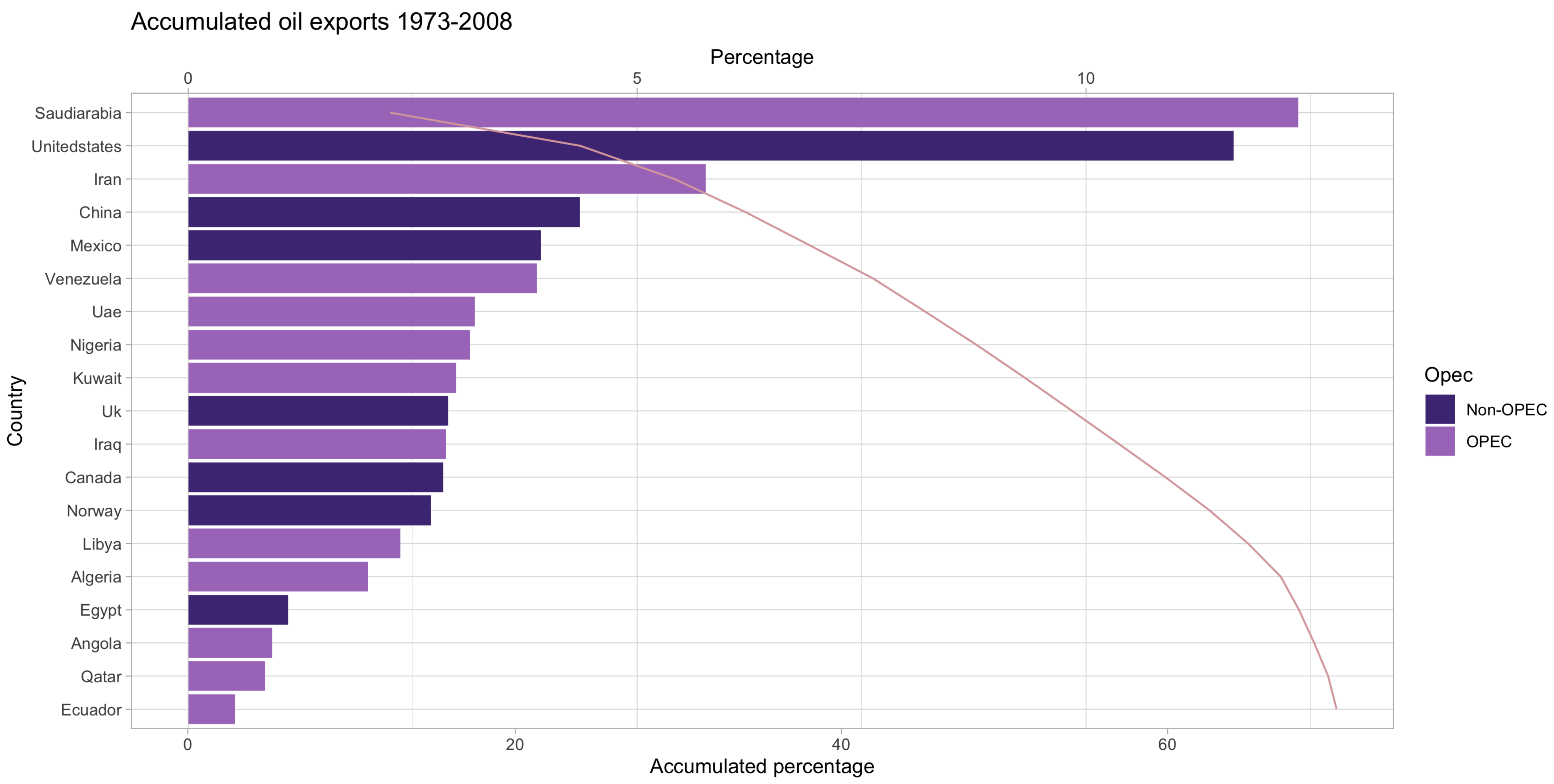
Assuming data $\{\mathbf{x}_i\}_{k=1}^{n}$ is i.i.d., hence, under the null hypothesis that the data is approximately consistent with SCR, it follows that:

$$\lim_{n\to\infty} \mathbb{P}\left(TS_n > \chi^2_{q,1-\alpha}\right) \leq \alpha \ \text{, for every } \alpha \in (0,1)$$

Data

- Production quantities data from the *Monthly Energy Review* (MER), published by the U.S. Energy Information Administration,
- MER provides a series of monthly crude oil production in thousands of barrels per day by 12 OPEC members and 7 nonmembers
- Price series come from the St. Louis Federal Reserve, dollars per barrel.
- The data covers January 1973 to April 2009. A total of 436 obs.

▶ Data table



Accumulated oil exports 1973-2008
Percentage
0
5
10
Saudiarabia
Unitedstates
Iran
China
Mexico
Venezuela
Uae
Nigeria
Kuwait
Uk
Iraq
Canada
Norway
Libya
Algeria
Egypt
Angola
Qatar
Ecuador
Country
0
20
40
60
Accumulated percentage
Opec
Non-OPEC
OPEC

Oil prices and exports

Results

	Big 6		OPEC		Non OPEC	
Time	TS	pvalue	TS	pvalue	TS	pvalue
1973-2008	0.0584590	0.9999959	0.0435110	0.9999983	0.0152957	0.9999999
1973-1980	0.0756257	0.9999912	0.2352666	0.9997516	0.1666667	0.9999094
1981-1990	0.0526316	0.9999970	0.0526316	0.9999970	0.3454260	0.9992452
1991-2000	0.2355584	0.9997507	0.2957727	0.9995173	0.2500009	0.9997035
2001-2008	0.4701321	0.9981832	0.0665860	0.9999940	0.0588235	0.9999959

Discussion and future work

This paper:

1. MC simulations to check if test has any bite
2. Unobserved price heterogeneity in the oil industry

Future projects:

1. Better demand estimates
2. Bertrand and product differentiation
3. Non-parametric Market Power test
4. Non-parametric bounds for marginal costs and markups
5. Reformulate using profit function

Using ELVIS

- In practice, we need to use Markov Chain Monte Carlo (MCMC) methods to compute $\tilde{h}(x;\gamma)$ by sampling from η and reject if the draw if it does not satisfy $1(g_R(x,\cdot)=0)1(g_C(x,\cdot)=0)$.
- In the application, I used a double hit-and-run algorithm adapted from Aguiar & Kashaev (2020) to sample directly from $\tilde{\eta}$.

Show 10 entries Search:

	Country	Total	Annual Avg.	%	OPEC
1	totalworld	27,079,707.0	731,884.0	100.0%	
2	totalnonopec	16,013,411.0	432,794.9	59.1%	
3	totalopec	11,066,301.0	299,089.2	40.9%	
4	saudiarabia	3,348,682.2	90,504.9	12.4%	OPEC
5	unitedstates	3,152,677.7	85,207.5	11.6%	Non-OPEC
6	iran	1,560,409.6	42,173.2	5.8%	OPEC
7	china	1,181,072.9	31,920.9	4.4%	Non-OPEC
8	mexico	1,063,022.5	28,730.3	3.9%	Non-OPEC
9	venezuela	1,051,849.4	28,428.4	3.9%	OPEC
10	uae	863,728.2	23,344.0	3.2%	OPEC

Showing 1 to 10 of 22 entries